THE CONTROVERSY ON THE ORIGIN OF OUR NUMERALS

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RECENTLY certain articles have been written which cast doubt upon the commonly accepted view that our numeral system originated in India and two writers definitely assign a European origin.[1] As the conclusions of these articles have been spread broadcast in popular weekly journals, it seems appropriate that a fuller account giving a digest of the facts and arguments bearing on the question be placed before the scientific public.

Our so-called "Arabic" notation owes its excellence to the application of the principle of local value and the use of a symbol for zero. It is now conclusively established that the principle of local value was used by the Babylonians much earlier than by the Hindus[2] and that the Maya of Central America used the principle and symbols for zero in a well-developed numeral system of their own.[3] The notation of Babylonia used the scale of 60, that of the Maya, the scale 20 (except in one step). It follows, therefore, that the present controversy on the origin of our numerals does not involve the question of the first use of local value and symbols for zero; it concerns itself only with *the time and place of the first application of local value to the decimal scale and with the origin of the forms or shapes of our ten numerals.*

[1] G. R. Kaye, "Notes on Indian Mathematics," *Journal and Proceedings of the Asiatic Society of Bengal*, N. S., Vol. 3, 1907, pp. 475–508; "The Use of the Abacus in Ancient India," *loc. cit.*, Vol. 4, 1908, pp. 293–297; "References to Indian Mathematics in certain Mediæval Works," *loc. cit.*, Vol. 7, 1911, pp. 801–813; "A Brief Bibliography of Hindu Mathematics," *loc. cit.*, Vol. 7, 1911, pp. 679–686; *Scientia*, Vol. 24, 1918, p. 54; "Influence Grecque dans le Développement des Mathématiques Hindoues," *Scientia*, Vol. 25, 1919, pp. 1–14.

Carra de Vaux, "Sur l'origine des chiffres," *Scientia*, Vol. 21, 1917, pp. 273–282.

Nicol. Bubnov, "Arithmetische Selbstständigkeit der europäischen Kultur," Berlin, 1914. (Translated from the Russian edition, Kiev, 1908.) "Origin and History of our Numerals," Kiev, 1908 (Russian).

[2] M. Cantor, "Vorlesungen über Geschichte der Mathematik," 1. Bd., 3. Auflage, Leipzig, 1907, pp. 24–43. Cantor gives bibliographical references.

[3] C. P. Bowditch, "Maya Numeration, Calendar and Astronomy," Cambridge (Mass.), 1910; S. G. Morley, "Introduction to the Study of the Maya Hieroglyphs," Washington, 1915.

That our numerals were of Hindu origin has been the belief held by individual European writers since the Renaissance. Following the publication of M. F. Woepcke's articles, particularly his "Mémoire sur la propagation des chiffres Indiens,"[4] it came to be generally accepted by mathematical historians. Only recently have dissenting voices been heard. Three writers, G. R. Kaye, Carra de Vaux, and Nicolaus Bubnov, represent the new claims. The last two writers place the weight of their authority on the side of an European origin.

The arguments upon which the Hindu origin of our numerals has been based are essentially three in number: (1) The use of the numerals in ancient Indian inscriptions, (2) the early Indian use of the abacus, (3) the testimony of Arabic writers.

G. R. Kaye, who, on this question, is far more careful, conservative and thorough than the other two investigators, has studied the Hindu numerals in connection with the general history of mathematics in India. He has made important contributions to this subject.

As regards the first argument, relating to ancient Hindu inscriptions, Kaye refers to seventeen inscriptions antedating the tenth century A.D. which have been supposed to contain our decimal place-value notation and to indicate the Indian origin of our numerals. The inscriptions are copper-plate grants. Many such grants are now known to be forgeries, fabricated about the end of the eleventh century, when there was "great opportunity to regain confiscated endowments and to acquire fresh ones." Students of epigraphy have eliminated from these seventeen inscriptions practically all but one as unauthentic, namely the one bearing the date 867 A.D.[5] Kaye states that the two earliest known Hindu inscriptions that contain complete sets of the ten numerals are of 1050 A.D. and 1114 A.D. According to the above, the earliest period of the undoubted use of our notation in India is the ninth century of our era. If the one inscription by which the ninth century is fixed turns out to be unreliable, then we must fall back on the tenth century as the earliest period.

Some writers have ascribed a knowledge of our notation to the astronomer Aryabhatta, early in the sixth century. L. Rodet[6] does so on the ground that Aryabhatta's rule for root-

[4] *Journal Asiatique*, 6 S., T. 1., Paris, 1863, pp. 27–79, 234–290, 442–529.

[5] G. R. Kaye, *Journal and Proceedings of the Asiatic Society of Bengal*, N. S., Vol. 3, 1907, pp. 485–487.

[6] *Loc. cit.*, p. 493.

extraction implied a use of the principle of local value. "Always divide the part that is not square by twice the root of the square, after having subtracted from this squared part the square of the root: the quotient is the root to the next term." Aryabhatta gives no illustrative examples. Rodet's inference does not follow, since the rule applies to all notations. Kaye points out that Theon of Alexandria gave such a rule, yet did not use a notation with place-value.

The second argument, that the early Hindus used the abacus, is rejected by Kaye, for the reason that there is no reliable evidence to support the claim. It has been held that it was the use of the abacus which, most likely, suggested the principle of local value.

The third argument, regarding the testimony of Arabic writers, reveals in some parts the strength of Kaye's contention of a non-Hindu origin and in other parts its weakness. Kaye shows conclusively that through a mistranslation, I. Taylor and M. F. Woepcke, and their followers, have ascribed to the Hindus the use of mathematical processes in early centuries, when, as a matter of fact, there is no evidence whatever to show that the Hindus actually used these processes at so early a date. This historical error arose according to Kaye in the mistranslation of the word *hindasi*. Woepcke admits that ordinarily this word signifies "geometrical," "measure," but asserts that this interpretation seemed impossible when used in connection with an explanation of the rule of "double false position" and the process of "casting out the nines," for the reason that these processes are purely arithmetical[7] in nature. Because of the resemblance of *hindasi* to the word *hindi* or "Indian," Woepcke concluded that with the particular authors in question *hindasi* meant "Indian," and that, therefore, the "double false position" and "casting out the nines" were known to the early Hindus. The latter would seem to imply the use of our notation. But Kaye was able to show that a geometrical interpretation of the passages in question was not only possible, but had actually been found in Arabic books.[8] Moreover, authorities on the Arabic language declare that *hindasi* can not mean *hindi*. Hence, says Kaye, Woepcke's inference that the early Hindus used the method of "double false position" and the process of "casting out nine" is wholly without foundation.

[7] M. F. Woepcke, *Journal Asiatique*, 6 S., T. 1., Paris, 1863, pp. 505, 511.

[8] G. R. Kaye in *Jour. and Proceed. of the Asiatic Society of Bengal*, Vol. 7, 1911, pp. 806–811.

Kaye admits that *hindi* means only "Indian" and that there are Arabic authors who speak of "Indian" numerals and methods of computation. Some light on the probable Hindu origin was obtained only a few years ago,[9] when a passage from the Mesopotamian scholar, Severus Sebokht, indicated that in the latter half of the seventh century the nine numerals were known in Arab lands and were attributed to the Hindus. Hurt by the alleged arrogance of certain Greek scholars, Sebokht praises the science of the Hindus and speaks of "their valuable methods of computation. . . . I wish only to say that this computation is done by means of nine signs." Unfortunately, he leaves it to us to guess whether or not he used the zero. The passage, written about 662 A.D., is the earliest reference that has been found outside of India to our numerals.

About two centuries after Sebokht, appeared the famous arithmetic of the Arab Alchowarizmi. The Arabic original is lost, but a Latin translation has come down to us under the title "Algoritmi de numero Indorum." While this title refers to Indian numerals, they are not actually used in the book. A book on the astronomical tables of Alchowarizmi that was written by Muhammed ibn Ahmed el-Bîrûnî (973–1038) was translated into Hebrew by Rabbi ben Ezra, who says in his introduction that a Hindu astronomical work had been translated into Arabic and that, after the time of Alchowarizmi, "scholars do their multiplications, divisions, and extraction of roots as is written in the book of the [Hindu] scholar which they possess in translation."[10] Other Arabic authors who in the titles of their texts refer to the Hindus are enumerated by Kaye.[11] Thus, about 987 A.D. appeared "The great Treatise on the Table relating to the Indian Calculus." Soon after came "The Principles of the Indian Calculus," and about 1030 "The satisfactory Treatise on Indian Arithmetic." There were two works, both bearing the same title, "Indian Arithmetic," one of the ninth century, the other of the tenth. A Latin text, attributed to Abraham, a Jew of whom little is known, is entitled "Liber augmenti et diminutionis vocatus numeratio divinationis, ex eo quod sapientes Indi posuerunt." The Italian Leonardo of Pisa, after traveling in Egypt, Syria, Greece, Sicily, wrote in 1202 his *Liber abbaci* in which he calls our

[9] M. F. Nau, *Journal Asiatique*, S. 10, Vol. 26, 1910; D. E. Smith and J. Ginsberg, *Bulletin Am. Math. Society*, Vol. 23, 1917, p. 368.

[10] See D. E. Smith, "Rabbi ben Ezra and the Hindu-Arabic Problem," *Am. Math. Monthly*, Vol. 25, 1918, p. 103.

[11] G. R. Kaye, *Journal and Proceedings of the Asiatic Society of Bengal*, N. S., Vol. 7, 1911, pp. 814–816.

numerals with the zero "figuræ Indorum." The Byzantine monk, Maximus Planudes (1260–1330), wrote an "Arithmetic according to the Hindus." The evidence from these and some other texts that we have omitted, in favor of the Hindu origin of our numerals, is not so strong as one might think. In some cases no Hindu symbols are actually employed by the authors; the arithmetic and algebra set forth do not seem to bear Hindu characteristics. Kaye suspects that the word "Indian" was often incorrectly applied. Yet this testimony, as a whole, comes with a force that is difficult to break.

Kaye has sought light on the history of our numerals in other studies. The successive units of our notation increase from right to left. Thus, we write the present year 1919, and not 9191. Therefore, our notation was probably invented by people with a right to left script and not by the Hindus whose script is from left to right. Kaye concedes that this argument is weakened by several considerations; thus, it is known that certain scripts have reversed their direction.

Again, Kaye points out that an "Old Indian" notation without the zero was used in India as late as the twelfth and thirteenth centuries. The form of the symbols with the zero, used in India, differed so widely from the old forms without the zero used there, that the former seem to have had an independent origin and to have been imported into India.

Let us now examine the arguments put forth by the Parisian scholar, Carra de Vaux. He quotes a well-known passage from the Arabic historian Masoudi writing in 943 A.D., giving a legend on creation which De Vaux recognizes as one due, no doubt, to the Neoplatonists in Persia.[12] This legend ascribes an Indian origin to our numerals. De Vaux's contention that the belief in the Indian origin, held by modern writers on the history of mathematics, rests simply upon this legend, is hardly in accordance with fact. Too indirect or circumstantial to be convincing is de Vaux's next point. He says that the Arabic author Albiruni (died 1038) must have drawn his information about Indian numerals from the above named legend, for otherwise he would not have given simply a general statement, but would have followed his usual custom of giving almost overscrupulously precise and detailed accounts.

We have seen that Woepcke erroneously attributed to the Arabic word hindasi the significance of *hindi* or "Indian," and consequently drew some wrong conclusions. De Vaux argues

[12] See Carra de Vaux in *Scientia*, Vol. 21, 1917, p. 274. The quotation from Masoudi is given in *Jour. and Proceed. of the Asiatic Soc. of Bengal*, N. S., Vol. 7, 1911, p. 812.

the other way, namely, that *hindi* does not mean "Indian," but means hindasi or "measure," "geometry," "arithmetic." Hence, when Arabic authors speak of *hindi* numerals, they do not mean "Indian numerals." The only support advanced for this unusual and strange interpretation is that an Arabic writer of the ninth century asks the question, "who is the inventor of the *hindi* figures," implying that he did not know the answer. It is possible that the question might have meant "who *in India* is the inventor of the hindi figures." De Vaux states that the Arabs did not ascribe the abacus to India; it is called *takht*, which is said to be Persian. De Vaux conjectures that the Arabs got the numerals with the zero from the Persians, who, in turn, got them from the Neoplatonists or Neopythagorians of Greece. On this hypothesis it is easier, he says, to explain the diffusion of numerals among the different nations than on that of a Hindu origin. From the Greeks they naturally spread to the Latins (Bœthius, fifth century) and Persians, and from the Persians to the Arabs and Hindus. From the Arabs the numerals passed to Spain, where Gerbert found them in the tenth century. De Vaux's suggestions as to the parts played by Bœthius and Gerbert do not seem to give proper weight to the numerous researches on the authenticity of manuscripts and are open to grave doubts. In fact, De Vaux and Nicolaus Bubnov entertain opposite views with regard to the geometry of Bœthius, particularly the part which contains the account of the nine numerals. Bubnov[13] concludes that it was written in the eleventh century, while De Vaux assigns it to the fifth. Bubnov gave a preliminary exposition of his hypothesis on the origin of our numerals in his 1899 edition of Gerbert's mathematical works. A fuller treatment followed in his book, "The Arithmetical Independence of European Culture," which appeared in Russian in 1908 and was translated into German in 1914. In the same year 1908 he issued in Russian another publication, "Origin and History of our Numerals," We have not enjoyed the opportunity of consulting his last work directly, but a rather full synopsis is given in the *Fortschritte der Mathematik*, 1908, pp. 53, 54. Philological studies lead Bubnov to deny the Hindu origin of our numerals, to claim that in the tenth to the twelfth centuries Europe possessed the modern positional arithmetic, though clothed in the form of the abacus with columns and marked reckoning counters. Bubnov holds that these counters marked with ancient symbols (the progenitors of our numerals) had superseded the older unmarked counters. He points out the existence of a counter which stood for zero (rotula supervacua) and claims that our modern Euro-

[13] See *Fortschritte der Mathematik*, Vol. 38, 1907, p. 62.

pean numerals have no connection with India. Thus he claims that Europe possessed the modern positional arithmetic in *instrumental* form, the instrument being an abacus with columns and marked reckoning counters. He asserts with confidence that the abacus with marked counters was used by the ancient Greeks and Romans, even though (as far as known) no such counter has come down to our time or has been described by writers of antiquity. He says that when *written* arithmetic supplanted instrumental arithmetic, the nine numerals and the zero, which first appeared on counters, finally descended upon the written page, but he has no evidence to support this admittedly clever hypothesis. Nor is he able to point to any European document which contains our nine numerals and the zero as early as they are found in India. Of course, Bubnov has a perfect right to set up hypotheses of his own,but his writings display an inclination on his part to parade unproved hypotheses in the guise of fairly well established facts. That his contentions should be viewed merely as unproved hypotheses appears also from the comments made by Sintzov,[14] Smith and Karpinski,[15] Paul Tannery[16] and G. Enestrom.[17]

SUMMARY

The following are the outstanding facts:

1. The earliest reliable record of the use of our numerals with the zero is an inscription of 867 A.D. *in India.*

2. The validity of the testimony of early Arabic writers ascribing to India the numerals with the zero is shaken, but not destroyed.

3. There is not a scintilla of evidence in the form of old manuscripts or numeral inscriptions to support the Greek origin of our numerals.

4. At present the hypothesis of the Hindu origin of our numerals stands without any serious rival. But this hypothesis is by no means firmly established.

As a by-product of the discussion of recent years we must admit that, on the evidence presented, the claim that our numerals and the zero were used in India as early as the fifth century must be abandoned; our earliest apparently reliable evidence belongs to the ninth century. We must also abandon the claim that the early Hindus used the abacus, the rule of "double false position," and the process of "casting out the nines." These corrections are due to G. R. Kaye.

[14] *Fortschritte der Mathematik,* Vol. 39, 1908, p. 54.

[15] Smith and Karpinski, "Hindu-Arabic Numerals," 1911, p. 65.

[16] P. Tannery in *Bibliotheca mathematica,* 3. Series, Vol. 1,1900, p. 286.

[17] G. Enestrom in *Bibliotheca mathematica,* Vol. 14, 1913–14, p. 355.